Winchester Math Curriculum Grade 4

Subject	Mathematics
Grade/Course	Grade Four
Unit of Study	Unit 7 - Updated Version - Reviewing and Extending Fractions, Decimals and Multi-digit Multiplication
Pacing	April
Unit Summary	Unit 7 reviews and extends skills and concepts in several areas that are foundational to the major work of fifth grade. Students refine their skills in equivalent fractions and comparing fractions. In multiplication students review strategies they have used to solve problems and they explore the standard multiplication algorithm.

Overarching Mathematical Practices

4.MP.1 Make sense and persevere in solving problems.
4.MP.2 Reason abstractly and quantitatively.
4.MP.3 Construct viable arguments and critique the reasoning of others.
4.MP.4 Model with mathematics.
4.MP.5 Use appropriate tools strategically.
4.MP.6 Attend to precision.
4.MP.7 Look for and make use of structure.
4.MP.8 Look for and express regularity in repeated reasoning.

Unit CT Core Content Standards

4.OA.A 3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.
4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

***Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.**
***Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.**

4.NF.A.1 Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

4.NF.A.2 Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as 1/2. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, =, or <, and justify the conclusions, e.g., by using a visual fraction model.
4.NF.C.5 Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.2*For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100*.
4.NF.C.6 Use decimal notation for fractions with denominators 10 or 100. *For example, rewrite 0.62 as 62/100; describe a length as 0.62 meters; locate 0.62 on a number line diagram*.
4.NF.C.7 Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and justify the conclusions, e.g., by using a visual model.
4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems. *For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor*.

"Unwrapped" Standards	
Skills	**Content**
Solve	Multi-step word problems with whole numbers
Use	● All four operations ● Mental computation ● Estimation strategies (including rounding) ● Strategies based on place value ● Properties of operations ● Decimal notation for fractions with denominators of 10 or 100
Interpret	Remainders
Represent	Problems using equations with a letter for the unknown quantity
Assess	The reasonableness of answers
Multiply	● Whole numbers up to four digits by one digit ● Two two-digit numbers
Illustrate and explain	Calculations with equations, rectangular arrays, and/or area models
Explain	● Why a fraction(a/b) is equivalent to fraction (n x a)/(n x b) with visual fraction models ● How the number and size of the parts differ even though the two fractions are the same size
Recognize and generate	Equivalent fractions

Compare	• Two fractions with different numerators and different denominators • To a benchmark • Two decimals to hundredths by reasoning about size
Recognize	Comparisons are valid only when the two fractions or decimals refer to the same whole
Record	Comparisons with >,=, or <
Justify	Conclusions using visual models
Express	A fraction with denominator of 10 as an equivalent fraction with denominator of 100
Add	Two fractions with respective denominators 10 and 100
Apply	Area and perimeter formulas for rectangles to solve problems

Essential Questions	Corresponding Big Ideas
1. How do we decide when to use an exact answer or an estimate? 2. How do models and pictures help us understand equivalent numbers and values? 3. What is mathematical reasoning?	1. The context of a problem situation determines whether an estimate or an exact answer is appropriate. 2. Physical tools/models and representations help to visualize, clarify and make meaning of equivalent numbers and values. 3. Mathematical reasoning is evaluating a problem situation and justifying the pathway to a solution.

Evidence of Learning - Assessment

	Interim Assessment	Additional Evidence of Learning
• Unit 7 - Pre Assessment Module 1, Session 1 • Unit 7 Post-Assessment - Module 4, Session 4	• Comparing Fractions Checkpoint -Module 2, Session 1 • Problems and Equations Checkpoint - Module 3, Session 4	Options • Exit Tickets Observational Assessments • Continue to use the work places from Units 5 and 6 and observe and evaluate students

Smarter Balanced Interim Assessment

Smarter Balanced General Scoring Rubrics - 4 Rubrics included - Score Pt 4 to Score Pt 1

· **Smarter Balanced Interim Blocks**

· Interim assessment blocks may be used for a variety of assessment purposes, including: pre/post, interim and formative (additional evidence of learning).

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· The Style Guide, which aligns with the expectations of Smarter Balanced Assessments, will support the creation of unit- and standard-aligned items for instructional use.

· The items on the interim assessments are developed under the same conditions, protocols, and review procedures as those used in the summative assessments. Therefore, they assess the same Common Core State Standards, adhere to the same principles of Universal Design in order to be accessible to all students, and provide evidence to support Smarter Balanced claims in mathematics and ELA/literacy. The interim assessment items are non-secure but non-public. This means that educators may view the items, however, they should not be made public outside of classroom, school or district.

· **Unit-aligned Smarter Balanced Interim Assessment Block (IAB)*:**

Interim Assessment Block - access through CSDE Assessment Portal

- IAB - Fractions

**Some interim blocks show clear, strong alignment to priority standards within the unit. Other blocks have been placed in one specific unit but could be aligned to the priority standards of several units. Blocks have been spread out over the course of all units for a more balanced approach to assessment throughout the school year. These interim blocks, used in partnership with the Style Guide, will support the creation of unit- and standard-aligned items for instructional use.*

Learning Plan

Researched-based Instructional Resources and Methods

Sequence of Instruction:
Number Talk/Number Corner → Problem + Investigations →Work Places → Math Forum*→Daily Practice or Home Connection

Bridges Number Corner - The focus areas of Number Corner Aligned to Unit 7 are:

- Fractions - Number Corner March/April
  - Adding and subtracting fractions and mixed numbers
  - Review fractions with denominators of 10 and 100
  - Equivalent fractions
  - Adding fractions with unlike denominators
  - Multiplying fractions by whole numbers
  - Converting fractions to mixed numbers
- Decimals- Number Corner March/April
  - Decimal notation
  - Adding tenths and hundredths
- Division - Number Corner March/April
  - Division story problems with and without remainders

Bridges - Whole Group, Small Group and Independent Problem Centered Activities

Module 1	Module 2	Module 3	Module 4
Problem + Investigation ● Sessions 2-7 Problem String ● None	Problem + Investigation ● Sessions 1-4 Problem String ● None	Problem + Investigation ● Sessions 1-5 Problem String ● None	Problem + Investigation ● Sessions 1-3 Problem String ● None

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Work Place ● Session 1 Math Forum ● None Daily Practice ● Sessions 1-7 Home Connection ● Sessions 2,4,6	Work Place ● Session 1 Math Forum ● None Daily Practice ● Sessions 1-4 Home Connection ● Sessions 1 + 4	Work Place ● Session 1-3 + 5 Math Forum ● None Daily Practice ● Sessions 1-5 Home Connection ● Sessions 2 + 4	Work Place ● Session 1 + 4 Math Forum ● None Daily Practice ● Sessions 1-4 Home Connection ● Sessions 1 + 3

Possible Misconceptions	Teacher Moves
1. Students think the numerator and denominator are separate values and have difficulty seeing them as a single value.	1. Have students place particular fractions on a number line and/or find fraction values on a number line.
2. Students use addition and subtraction instead of multiplication to develop sets of equivalent fractions.	2. Provide students additional experiences with visual representations including fraction bars, area models, and the number line. Explanations of why one multiplies or divides to find an equivalent fraction should begin with visual representations and eventually connect to the rule/algorithm.
3. Students think that when generating equivalent fractions they need to multiply or divide either the numerator or denominator, such as, changing $\frac{1}{2}$ to sixths. They would multiply the denominator by 3 to get $\frac{1}{6}$, instead of multiplying the numerator by 3 also. Their focus is only on the multiple of the denominator, not the "whole fraction".	3. It's important that students use a fraction in the form of one such as $\frac{3}{3}$ so that the numerator and denominator do not contain the original numerator or denominator.
4. Students may believe that the greater the denominator, the larger the fraction. These students do not understand that a larger denominator represents a whole that has been divided into more and smaller equal sized pieces.	4. Use area models of fractions to illustrate the magnitude of the fraction in relation to the whole
5. Students who forget that the larger the number in the denominator, the smaller the piece, may base their comparisons on incorrect notions.	5. These students need additional practice with concrete models and making connections to the written numerals.
6. Students may believe that all halves are equal or that if fractions have the same name, they are equal. This misunderstanding arises when students	6. This will need to be addressed by illustrating the misconception with number lines and area models.

don't understand that comparing fractions must be done in relation to the same sized whole. Students may believe this because when comparing whole numbers, 2 is equal to 1+1. 7. Students treat decimals as whole numbers when making comparison of two decimals. They think the longer the number, the greater the value. For example, they think that .03 is greater than 0.3. 8. Students may ignore place value when multiplying multi-digit numbers. 9. Watch for students who get the place value of digits confused when dividing.	7. Use visual models for decimals such as base ten blocks or money. Using our money system, where the dime represents tenths and the penny represents hundredths, students may more easily see decimals as parts of a whole, with the whole being one dollar. Decimal fractions such as 45/100 can be easily modeled using dimes and pennies as 4 dimes and 5 pennies. 8. Use concrete materials to review place value understanding. (multiplying by a multiple of ten will give a product that is expressed as tens; 6 x 70 = 6 x 7 tens = 42 tens = 420) The use of partial products and the distributive property will help to reinforce each part of a multiplication equation. 9. Use the relationship between multiplication and division and students' previous experiences with estimation to help students realize the place value of the quotient.

Vocabulary and Representations	
Tier 2 (Academic Vocabulary)	**Tier 3 (Domain Specific Vocabulary**
area* compare dimension estimate* greater than* less than* meter* product* strategy total unknown	area model* area model of multiplication* array* centimeter* common denominator decimal* decimal fraction decimal number decimeter denominator* equivalent fractions* equation* fraction* hundredth*

*Smarter Balanced Vocabulary is focused on major mathematical concepts. (Not all possible words have been identified by SBAC) + Students are not responsible for these vocabulary words at this grade level, however they should have some understanding of the mathematical concept.	multiplication multiply* numerator* partial products* repeated addition square centimeters* square foot* standard algorithm sum* tenth*

Mathematics Teaching Practice Resources

1. **Bridges** - Reference Math Practices in Action Notes - The notes identify how particular mathematical practice is employed in a specific activity.
2. Math Practices Teacher Question Starters
3. Implementing the Standards of Mathematics Practice
4. Illustrating the Standards of Mathematical Practice
5. Math Practice Standards Posters Gr. 4-5
6. Grade 4 - Standards + Practices - Explanations and Examples
7. Number Talks Matter - Number Talks at a Glance and Fluency without Fear
8. Use and Connect Mathematical Representations
9. Pose Purposeful Questions
10. Mistakes are Powerful - Resource to develop students' perseverance through mistakes

11. Fractions and Decimals
    a. Number-*Fractions toward bottom of page*

    b. Learn Zillion - Grade 4

    c. Illustrative Math-Progression of Fractions Videos:
        i. 1. Meaning of Unit Fractions
        ii. 2. Equivalent Fractions
        iii. 3. Comparing Fractions
        iv. 4. Addition of Fractions
        v. 5. Multiplication of Fractions, Part 1

    d. Modeling of...
        - Strategies:
            - visual fraction models
            - benchmark fractions
            - decimal connections to money (dollars to whole numbers, dimes to tenths, and pennies to hundredths)
            - visual decimal models
            - equivalent fraction models
            - Quick Images - Fractions Area Model
            - Greg Tang Math-Satisfraction:
12. Virtual Manipulatives
13. Fraction Land
14. Journal Prompts for Math

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15. Bridges Interactive Math Manipulatives
16. Illustrative Math – Grade 4 - Resources and activities for the grade aligned by standard.
17. Inside Mathematics Multiplication Activity- The Baker Performance Task
18. Bridging Practices - UCONN – Training and a Task Repository to develop and support student capacity for argumentation in mathematics
19. Accountable Talk Moves
20. Contribution Checklist
21. Sentence Frames that Can Build Metacognitive Thinking
22. Sample Language Frames for Mathematics
23. Building a Mathematical Mindset Community
24. Teacher/Student Actions
25. Fletcher Three Act Tasks
26. Vocabulary Development Frayer Model – Elementary and secondary video and resources.
27.

Suggestions for Differentiation, Scaffolding and Intervention

Differentiation or Intervention

Any teacher moves/strategies that address misconceptions can be used in differentiation or as intervention.

Math Teaching Practice Resources contain resources that provide opportunities for differentiation, intervention, or extension aligned to the strategies below.

- How to Select Math Intervention Content
- Coherence Map in Math – The coherence map shows how standards within and across grades build upon each other. You can use the map to assist you in to build student understanding by linking together concepts within and across grades and identify gaps in a student's knowledge by tracing a standard back through its logical prerequisites.
- CT Dept. of Education Evidence-based Practice Guides – These guides provide links to "evidence-based activities, strategies and interventions (collectively referred to as 'interventions')."
- Evidenced-based strategies for supporting struggling students (U.S. Dept. of Education – What Works Clearinghouse)
- Ensure instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think alouds.
- Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.
- Teach students about the structures of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.
- Early childhood students should work with concrete representations and then visual representations of mathematical ideas.
- If visual representations are not sufficient for developing accurate abstract thought and answers, use concrete manipulative first.
- Provide students with workplace sentence frames
- Concrete, Representational, Abstract Progression
- Break multi-step word problems down into individual steps.

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- All students will not be ready at the same time to discontinue use of the base-ten manipulatives. The teacher will need to be conscientious about monitoring each student's level of understanding to know when the student will be ready to transition to diagrams without manipulatives support. If students experience frustration or uncertainty during this process, the manipulatives should be kept available for use as reinforcement or as a way to check the diagram.
- Difficulty with place value of whole numbers may lead to difficulty and confusion with decimals. Provide many concrete experiences modeling ones and tenths before moving to hundredths.
- Scaffold experiences so students are progressing from tenths to hundredths.
- Ask questions that support that support seeing the relationship between the tenths and hundredths places and between decimal places and one whole.
- As students model, reinforce conceptual understanding with question such as:
    - In the decimal 3.42, how many wholes (ones) are there?
    - How many tenths?
    - How many hundredths?
    - Why do we read the decimal as "three and forty-two hundredths" when there are only two hundredths?
- When working with decimals make sure they understand that decimals are another way to write fractions with denominators that are powers of ten (10ths, 100ths, 1000ths)
- Use money or a familiar context to help to build the understanding of the relationship of decimal to one whole.
- Compare decimals that are hundredths with decimals that are tenths using a strategy that makes sense (using fraction numbers, base-ten blocks, or the number line).
- Lack of conceptual understanding of division can impact the appropriate solution when remainders are involved. For example, 16 students are going canoeing. If each canoe holds 3 students, how many canoes will they need? The answer 5 r 1 makes no sense in this situation. Help students focus on the question and reasonableness of solutions using strategies including models, pictures, and acting out.
- Revisit comparison activities and include pairs in which the fractions are equal but do not appear to be.
- The general approach to helping students create an understanding of equivalent fractions is to have them use models to find different names for a fraction. Consider that this is the first time in their experience that a fixed quantity can have multiple names. Use area model for fractions that is familiar to the student, prepare two or three outlines of different fractions. The student's task is to use their own fraction pieces to find as many single fraction names for the region as possible. After they complete the three examples, have students write about the ideas or patterns they may have noticed in finding the names.
- Work Places Sentence Frames - Bridges Grade 4 Resource (Can be a live link with purchase of program).
- At times, partner them with students who are very articulate about their mathematical thinking so they can hear (through conversations) how these students have made sense of the problems.
- Have students create base-ten models on a place value mat and put the decimal notation directly underneath the model using the place-value chart.
- Allow students to refer to a meter stick while working on number lines. Each decimeter is one tenth of a meter and each centimeter is one hundredth of a meter

Intervention for facts

- Provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts. Consider using technology, flashcards, and other materials for extensive practice to

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facilitate automatic retrieval.

- For students in K -2 explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.
- Teach students in grade 2-8 how to use their knowledge of properties, such as commutative, associative, and distributive to derive facts in their heads.
- How to Promote Acquisition of Math Facts – Intervention for struggling students
- National Center on Intensive Intervention - Basic Facts
- Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises should be varied in type and focus as much on the discussion of how students obtained their answers as on the answers themselves.
- Having students work in groups (as opposed to handing your bright students a workbook to work on when the classroom material isn't challenging enough) with other children ready for advanced material shows them that mathematics is not a solitary discipline -- mathematics is exciting and vibrant and creative and fun.
- Struggles with basic facts - need more experience with concrete and pictorial representations, including describing what their models represent to make connection to basic facts. Time and experience with developing strategies that are based on patterns and properties will help support learning the facts. It is important to give students time to learn and understand these concepts before procedural skill practice takes place.

El Strategies

- Colorin Colorado – A Bilingual site for educators and families of English learners
- Stanford University - Principles for Mathematics Instruction of EL
- CT State Dept. Of Education English Learner Standards and Resource
- Nonverbal responses, such as thumbs up, will help you check for understanding without requiring students to produce language. ELLs can participate and show that they understand a concept, or agree or disagree with an idea, without having to talk. This is especially important for students whose comprehension of English is more advanced than their ability to speak the language.
- Pre-teach vocabulary in ways that connect to students prior knowledge.
- Display posters of graphic representations of vocabulary words.
- Provide support to assist in explaining thinking with sentence starters and work banks.
- Use Work Place Sentence Frames or other sentence frames to assist students in math discourse.
- Speak slowly and use clear articulation. Reduce the amount of teacher talk and use a variety of words for the same idea. Exaggerate intonation and place more stress on important new concepts or questions. After asking a question, wait for a few moments before calling on a volunteer. Writing the question on the board will also help.
- English language learners are not always able to answer the questions posed to them, especially when the questions are open-ended. Provide support for and improve the participation of students with lower levels of English proficiency by using a prompt that requires a physical response, like "Show me a half, a third, etc.." or "Touch the larger number."
- http://www.cal.org/siop/lesson-plans/
- Speak slowly and use clear articulation. Reduce the amount of teacher talk and use a variety of words for the same idea. Exaggerate intonation and place more stress on important new concepts or questions. After asking a question, wait for a few moments before calling on a volunteer. Writing the question on the board will also help.
- Increase academic language knowledge for English learner success.

Extension

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- Have children half, double and/or triple a recipe and estimate how many people the new recipe will feed.
- Using a 10 x 10 frame, for each of the following fractions determine the decimal equivalent and explain your reasoning.
  - $\frac{3}{4}$ $\frac{2}{5}$ $\frac{3}{8}$
  - Shade in the fractional amount
  - Identify the decimal number that also represents this amount
  - Explain your reasoning
- ask students to create and solve their own story problems
- Ask students to explain why making other fractions such as 1/7 would be difficult. Can you name other fractions that would be difficult? How would you go about making them?
- Students could explore denominators other than those listed in the standard, such as 9, 15, 24, etc.
- Encourage students to try a different strategy to determine if they will arrive at the same conclusions when working with fractions.
- Compare and order more than two fractions.
- Have students label mixed numbers as an improper fractions as well.
- Have students create their own new model/representation for wholes, tenths, and hundredths and use these models to draw decimal fractions. Students should label their models with decimal notation.
- Students can be encouraged to conduct a survey of 10 people or 100 people and report the results as decimal fractions.
- Provide students all types of coins and/or bills in the bag of money and have them change all coins into “decimal fraction” friendly coins and justify the exchanges.
- Allow students who are ready to explore thousandths.
- Give students two numbers, for example 3.2 and 3.3. Ask students to list at least 9 numbers that come between these two numbers (3.21, 3.22, 3.23, 3.24...3.29). Ask students if they think there are numbers between 3.21 and 3.22.

Interdisciplinary Connections

Children’s Literature - * Bridges recommended titles - # Titles embedded in Bridges Units

A Remainder of One by E.J.Pinczes
What’s Cooking, Jenny Archer by Ellen Conford
Gator Pie by Louise Matthews
Working with Fractions, David A. Adler
Cookies by William Jaspersohn
Divide and Ride by Stuart J. Murphy
Alexander Who Used to Be Rich Last Sunday by Judith Viorst
The Man Who Counted: A Collection of Mathematical Adventures by Tahan
Polar Bear Math: Learning About Fractions from Klondike and Snow by Ann Whitehead Nagda
*The Hershey’s Milk Chocolate Fraction Book by Jerry Pallotta
*Fraction Action by Loreen Leedy

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Arts
Use mathematics as a way to arrange visual characteristics

Writing
Make generalizations and/or conjectures and justify conclusions using evidence.

Science

- Describe, measure, estimate, and/or graph quantities such as area, volume, weight, and time to address scientific and engineering questions and problems
- Mathematics terminology accurately used in science activities such as, measuring using fractions and/or decimals.
- Accurate use of calculations in science