Winchester Math Curriculum Grade 4

Subject	Mathematics
Grade/Course	Grade 4
Unit of Study	Unit 2: Multi-Digit Multiplication & Early Division
Pacing	October
Unit Summary	Students continue to build multiplicative reasoning as they work with multi-digit multiplication and early division. In Module 1, students use base ten area pieces to investigate place value patterns, as well as model and solve single- and double-digit multiplication problems. In Module 2, they move from building multiplication arrays to sketching them. They also build ratio tables and use them to make generalizations about the effects of multiplying by 10, 100, and 1,000. In the third module, students solve a variety of multiplication story problems, and work together to compile and compare the strategies they have been practicing. In Module 4, students solve division problems that require them to make sense of remainders in a variety of contexts. New Work Places provide more practice with multiplication facts, multi-digit multiplication, and division with remainders.

Overarching Mathematical Practices

4.MP.1   Make sense and persevere in solving problems.
4.MP.2   Reason abstractly and quantitatively.
4.MP.3   Construct viable arguments and critique the reasoning of others.
4.MP.4   Model with mathematics.
4.MP.5   Use appropriate tools strategically.
4.MP.6   Attend to precision.
4.MP.7   Look for and make use of structure.
4.MP.8   Look for and express regularity in repeated reasoning.

Unit CT Core Content Standards

4.OA.A.3 Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quality. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

4.OA.B.4 Find all factor pairs for a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1-100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1-100 is prime or composite.

4.NBT.A.1 Recognize that in a multi-digit whole number, a digit in one place represents ten time what it represents in the place to its right.

4.NBT.A.2 Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, =, and < symbols to record the results of comparisons.

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4.NBT.B.5 Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit number, using strategies based on place value and the properties of operations. Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models.

4.NBT.B.6 Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculations by using equations, rectangular arrays, and/or area models.

4.MD.A.1 Know relative sizes of measurement units within one system of units including km, m, cm; kg, g;lb, oz.;l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two-column table.

4.MD.A.2 Use the four operations to solve word problems involving distance, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.

4.MD.A.3 Apply the area and perimeter formulas for rectangles in real world and mathematical problems.

"Unwrapped" Standards	
Skills	**Content**
Solve	multi-step word problems with whole numbers and whole number answers
Interpret	remainders
Represent	- problems using equations with a letter for the unknown - measurement quantities using diagrams
Assess	the reasonableness of answers
Find	all factor pairs for a whole number
Determine	- whether a given whole number (1-100) is a multiple of a given one-digit number - whether a given whole number is prime or composite
Recognize	in a multi-digit whole number a digit in one place represents ten times what it represents in the place to its right
Multiply	four digits by one digit and twoby two-digit numbers using strategies
Illustrate and Explain	calculations by using equations, rectangular arrays, and/or area models
Find	whole number quotients and remainders with up to four-digit dividends and one digit divisors using

	strategies
Know	relative sizes of measurement units within one system
Express	within a system measurements in a larger unit is terms of a smaller unit
Record	measurement equivalents in a two-column table
Use	four operations to solve word problems involving measurement
Apply	area and perimeter formulas for rectangles in real world and mathematical problems

Essential Questions	Corresponding Big Ideas
1. Why is estimation an important tool?	1. Estimation is an important life skill that people use everyday. Many real-life applications of math do not require exact answers. The problem situation determines the best estimation strategy to use. Estimation is also an effective strategy that promotes easy recognition of the reasonableness of an answer and for catching errors made when using calculators.
2. How can I become an effective and efficient problem solver?	2. Finding common characteristics among similar problems and understanding that each problem is a member of a larger class of problems will help in identifying the most efficient and effective strategies to solve problems.
3. Why is it important to understanding how to find the factors of a number?	3. Factoring numbers can help build computation fluency. Breaking down a number into its factors can be utilized as an efficient strategy in variety of computational situations.
4. How can understanding place value aid computation?	4. In the base ten number system place value is based on groups of ten. Understanding that each place to the right increases by ten times and moving left the places decrease by a division of ten or 1/10th.

Evidence of Learning - Assessment

Pre/Post Assessment	Interim Assessment	Additional Evidence of Learning
● Unit 2 Pre-Assessment & Student Self-Reflection - Module 1, Session 2 ● Unit 2 Post-Assessment & Student Self-Reflection - Module 4, Session 5 ● Number Corner Checkup 1	● Multiplying by Tens Checkpoint - M2, S5 ● Multiplication Checkpoint - M3, S3 ● Multiplication Strategies Work Sample - M3, S5 ● Number Corner Checkup 1	Options ● Exit tickets Observational Assessments ● What's Missing? M1, S4 ● Division Capture - M2, S1 ● Moolah on My Mind - M3, S4

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			● Remainders Win - M4, S3 ● More or Less Multiplication - M4, S4

Smarter Balanced Interim Assessment

Smarter Balanced General Scoring Rubrics - 4 Rubrics included - Score Pt 4 to Score Pt 1
· **Smarter Balanced Interim Blocks**

· Interim assessment blocks may be used for a variety of assessment purposes, including: pre/post, interim and formative (additional evidence of learning).

· The Style Guide, which aligns with the expectations of Smarter Balanced Assessments, will support the creation of unit- and standard-aligned items for instructional use.

· The items on the interim assessments are developed under the same conditions, protocols, and review procedures as those used in the summative assessments. Therefore, they assess the same Common Core State Standards, adhere to the same principles of Universal Design in order to be accessible to all students, and provide evidence to support Smarter Balanced claims in mathematics and ELA/literacy. The interim assessment items are non-secure but non-public. This means that educators may view the items, however, they should not be made public outside of classroom, school or district.

· **Unit-aligned Smarter Balanced Interim Assessment Block (IAB)*:**
Interim Assessment Block - access through CSDE Assessment Portal
   ● IAB - Number and Operations in Base Ten

· *Some interim blocks show clear, strong alignment to priority standards within the unit. Other blocks have been placed in one specific unit but could be aligned to the priority standards of several units. Blocks have been spread out over the course of all units for a more balanced approach to assessment throughout the school year. These interim blocks, used in partnership with the Style Guide, will support the creation of unit- and standard-aligned items for instructional use.*

Learning Plan

Researched-based Instructional Resources and Methods

Sequence of Instruction:
Number Talk/Number Corner → Problem + Investigations →Work Places → Math Forum*→Daily Practice or Home Connection

Bridges Number Corner - The focus areas of Number Corner Aligned to Unit 2 are:
   ● Fractions and Decimals - tenths, fifths, fourths, and halves
       ○ as parts of a dollar
       ○ on a base ten mat
       ○ in a divided up rectangle and/or pentagon
   ● Place Value
       ○ to millions
       ○ rounding and comparing numbers
       ○ writing numbers in different forms

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- Work with Multiples
  - add multiples of 4 and 8
  - record multiples on classroom number line
  - explore relationships between factors and multiples
- Multi-digit addition and subtraction
  - Rounding
  - Estimating
  - understanding place value
- Multiplication/Multiplicative Thinking
  - using ratio tables
  - partial products
  - doubling
  - proportional reasoning
- Solve problems
  - multi-step with multiplication
  - breaking complex problems into a series of smaller steps
  - discuss strategies and solutions
  - error analysis
  - make generalizations on how to avoid errors on multi-step problems

Bridges - Whole Group, Small Group and Independent Problem Centered Activities

Module 1	Module 2	Module 3	Module 4
Problem + Investigation ● Sessions 1, 3, 4, 5 Problem String ● None Work Place ● Sessions 2, 4 Math Forum ● None Daily Practice ● Sessions 1-5 Home Connection ● Sessions 2, 4	Problem + Investigation ● Sessions 1-5 Problem String ● None Work Place ● Sessions 1, 2, 4 Math Forum ● None Daily Practice ● Sessions 1-5 Home Connection ● Sessions 1, 3, 5	Problem + Investigation ● Sessions 1, 4, 5 Problem String ● Session 3 Work Place ● Sessions 2, 3, 4, 5 Math Forum ● Session 2 Daily Practice ● Sessions 1-5 Home Connection ● Sessions 2, 4	Problem + Investigation ● Sessions 1, 2, 4 Problem String ● None Work Place ● Sessions 1, 3, 4, 5 Math Forum ● Session 3 Daily Practice ● Sessions 1-5 Home Connection ● Sessions 1, 3, 5

Possible Misconceptions	Teacher Moves

1. Students who struggle in determining what operation to use to solve a problem.	1. These students need additional experience understanding the operations in a variety of situations. They should have explicit practice with various problem solving strategies, including:
	   a. restating the problem in their words
	   b. Identifying given, needed and wanted

information
   c.   making a model or drawing a picture
   d.   making a list
   e.   acting it out
   f.   finding a pattern
   g.   writing an equation
   h.   revisiting the equation and asking themselves if the solution makes sense

2. Students who do not have conceptual understanding or have focused work on rote division procedures tend to write answers to problems using the "r" notation for remainder, even if it makes no sense with the context of problem. For example, 16 students are going canoeing. If each canoe holds 3 people, how many canoes will they need?

2. Students need to focus on the question and reasonableness of solutions using strategies including models, pictures, and acting out.

3. Students may become confused with extending place value patterns and focus on zeros rather then the value of the digit based on its place such as, 700 is ten times 70 and 70 is ten times 7 or 7 is one-tenth of 70 or 70 divided by 10 to demonstrate that a digit in one place represents ten times what it represents in the place to its right.

3. Concrete models comparing ones, tens, and hundreds and using the appropriate language (ten times greater, one-tenth or divided by ten) will help students recognize, extend and describe patterns based on understanding rather than what the number looks like.

4. Student may ignore place value when multiplying multi-digit numbers.

4. Use concrete materials to review place value understanding (multiplying by a multiple of ten will give a product that is expressed as tens: 6 x 70 = 6 x 7 = 42 tens = 420). At this time the use of partial products and the distributive property will help to reinforce each part of a multiplication equation. Extending simple area models to area models for multi-digit multiplication examples will also reinforce the role of partial products.

5. Watch for students who get the place value of digits confused when dividing.

5. Use the relationship between multiplication and division and students previous experiences with estimation to help them realize the place value of the quotient.
   a.   Consistently model questions such as the following to help students hone in on the quotient. Example: Martin has 183 Hot Wheels cars in his collection. He has boxes that each hold 8 Hot Wheels cars. How many boxes will he need to store the cars?
      ■   Can he fill 10 boxes? Yes. How many cars will 10 boxes hold? 80 cars

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■ Can he fill 20 boxes? Yes. How many cars will 20 boxes hold? 160 cars
■ Can he fill 30 boxes? No. How many cars would 30 boxes hold? 240 cars
■ So if 20 boxes will hold 160 cars, how many cars still need to be put in a box? 23 cars
■ How many boxes will be full? 20+2=22 boxes
■ Are there any cars that are not in a box? 1 car
■ If he wants to put all of the cars in a box, how many boxes will he need? 23 boxes

b. Use partial quotients to allow student to chunk numbers into smaller pieces, make the problem more manageable, and avoid mistakes when there is a zero in the quotient.
c. Give students opportunities to find and use compatible numbers in determining the quotient.

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Vocabulary and Representations

Tier 2 (Academic Vocabulary)	Tier 3 (Domain Specific Vocabulary
area*	area model*
dimension	array*
divide*	associative property of multiplication
division*	benchmark number
double (doubling)	centimeter (cm)*
estimate*	dividend
estimation*	divisor
factor*	equation*
grouping	expanded form
pattern*	gram (g)*
product*	hundred*
relationship	hundred thousands*
remainder*	kilogram (kg)*
rounding*	landmark number
strategies	liter (l)*
unit	meter (m)*
unknown	millimeter (ml)*
	numeral
*Smarter Balanced Vocabulary is focused on major	partial products*
mathematical concepts. (Not all possible words have been	place value*

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identified by SBAC) + Students are not responsible for these vocabulary words at this grade level, however they should have some understanding of the mathematical concept.	quotient* ratio table (table of equivalent ratios) square centimeters* ten thousand* thousands* yard (yd)

Mathematics Teaching Practice Resources

1. **Bridges** - Reference Math Practices in Action Notes - The notes identify how particular mathematical practice is employed in a specific activity.
2. Illustrating the Standards for Mathematical Practice
3. Grade 4 Mathematical Practice Question Stems
4. Math Practice Standards Posters Gr. 4 -5
5. **Implementing the Standards of Mathematics Practice**
6. Multiplication and Division Problem Situation Types
7. Bridges - Interactive Math Manipulatives
8. Use and Connect Mathematical Representations
9. Bridging Practices - UCONN – Training and a Task Repository to develop and support student capacity for argumentation in mathematics
10. Base Ten Place Value - Students are given opportunities to explain the relationship between place value and the power of ten in multiplying and dividing.
11. Accountable Talk Moves
12. Contribution Checklist
13. Sentence Frames that Can Build Metacognitive Thinking
14. Sample Language Frames for Mathematics
15. Building a Mathematical Mindset Community
16. Teacher/Student Actions
17. Illustrative Math
    a. Thousands and Millions of Fourth Graders
    b. Mental Division Strategy - Critiquing the reasoning of others.
    c. Comparing Growth- Variation 1 - Solving problems
    d. Comparing Growth - Variation 2 - Solving problems
18. LearnZillion
    a. Model Number using Base Ten Blocks
    b. Understand the Relationship between Digits and their Place Value
    c. Multiply Multi-digit Whole Numbers by Single Digit Whole Numbers Using an Area Model
    d. Multiply 2-Digit Whole Number Multiples of Ten by 2-Digit Multiples of Ten Using the Area Model
    e. Solve Multi-Digit Multiplication Problems by Using Place Value Understanding
    f. Use Place Value Understanding to Solve Division Problems involving up to 4 Digit Dividends by 1 Digit Divisors that have Remainders

Suggestions for Differentiation, Scaffolding and Intervention

Differentiation or Intervention
Any teacher moves/strategies that address misconceptions can be used in differentiation or as interventions.
Math Teaching Practice Resources contain resources that provide opportunities for differentiation,

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intervention, or extension aligned to the strategies below.

- [How to Select Math Intervention Content](#)
- [Coherence Map in Math](#) – The coherence map shows how standards within and across grades build upon each other. You can use the map to assist you in to build student understanding by linking together concepts within and across grades and identify gaps in a student's knowledge by tracing a standard back through its logical prerequisites.
- [CT Dept. of Education Evidence-based Practice Guides](#) – These guides provide links to "evidence-based activities, strategies and interventions (collectively referred to as 'interventions')."
- Evidenced-based strategies for supporting struggling students (U.S. Dept. of Education – [What Works Clearinghouse)](#)
- Ensure instructional materials are systematic and explicit. In particular, they should include numerous clear models of easy and difficult problems, with accompanying teacher think alouds.
- Provide students with opportunities to solve problems in a group and communicate problem-solving strategies.
- Teach students about the structures of various problem types, how to categorize problems based on structure, and how to determine appropriate solutions for each problem type.
- Students should work with visual representations of mathematical ideas.
- Difficulty with place value of whole numbers may lead to difficulty and confusion with decimals. Provide many concrete experiences modeling ones and tenths before moving to hundredths.
- When working with decimals make sure they understand that decimals are another way to write fractions with denominators that are powers of ten (10ths, 100ths, 1000ths)
- If visual representations are not sufficient for developing accurate abstract thought and answers, use concrete manipulative first. (Include the next line for middle school and older students only) Although this can also be done with students in upper elementary and middle school grades, use of manipulatives with older students should be expeditious because the goal is to move toward understanding of and facility with visual representations and finally to the abstract.
- Any teacher moves/strategies that address misconceptions can be used in differentiation or as interventions.
- [Comparing Growth- Variation 1](#) - Solving problems
- [Comparing Growth - Variation 2](#) - Solving problems
- Provide carefully constructed questions to help direct students in determining what to do to solve problems, but they shouldn't be told how to reach the solution.
- Instruction during the intervention should be explicit and systematic. This includes providing models of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, and frequent cumulative review.

Intervention for facts

- Provide about 10 minutes per session of instruction to build quick retrieval of basic arithmetic facts. Consider using technology, flashcards, and other materials for extensive practice to facilitate automatic retrieval.
- For students in K -2 explicitly teach strategies for efficient counting to improve the retrieval of mathematics facts.
- Teach students in grade 2-8 how to use their knowledge of properties, such as commutative, associative, and distributive to derive facts in their heads.
- [How to Promote Acquisition of Math Facts](#) – Intervention for struggling students
- [National Center on Intensive Intervention - Basic Facts](#)
- Once a strategy has been taught, it is important to reinforce it. The reinforcement or practice exercises

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should be varied in type and focus as much on the discussion of how students obtained their answers as on the answers themselves.
- Having students work in groups (as opposed to handing your bright students a workbook to work on when the classroom material isn't challenging enough) with other children ready for advanced material shows them that mathematics is not a solitary discipline -- mathematics is exciting and vibrant and creative and fun.
- Struggles with basic facts - need more experience with concrete and pictorial representations, including describing what their models represent to make connection to basic facts. Time and experience with developing strategies that are based on patterns and properties will help support learning the facts. It is important to give students time to learn and understand these concepts before procedural skill practice takes place.
- Concrete, Representational, Abstract Progression

EL Strategies
- Colorin Colorado – A Bilingual site for educators and families of English learners
- Stanford University - Principles for Mathematics Instruction of ELs
- CT State Dept. Of Education English Learner Standards and Resources
- Nonverbal responses, such as thumbs up, will help you check for understanding without requiring students to produce language. ELLs can participate and show that they understand a concept, or agree or disagree with an idea, without having to talk. This is especially important for students whose comprehension of English is more advanced than their ability to speak the language.
- Pre-teach vocabulary in ways that connect to students' prior knowledge.
- Display posters of graphic representations of vocabulary words.
- http://www.cal.org/siop/lesson-plans/
- Provide support to assist in explaining thinking with sentence starters and work banks.
- Use Work Place Sentence Frames or other sentence frames to assist students in math discourse.
- Speak slowly and use clear articulation. Reduce the amount of teacher talk and use a variety of words for the same idea. Exaggerate intonation and place more stress on important new concepts or questions. After asking a question, wait for a few moments before calling on a volunteer. Writing the question on the board will also help.
- English language learners are not always able to answer the questions posed to them, especially when the questions are open-ended. Provide support for and improve the participation of students with lower levels of English proficiency by using a prompt that requires a physical response, like "Show me a half, a third, etc.." or "Touch the larger number."
- Increase academic language knowledge for English learner success.

Extensions
- Mathwire Problem Solving Opportunities
- Harvard Balanced Assessment Project Tasks - Tasks can be used for purposes other than assessment
- Math Solutions - The Game of Target

Interdisciplinary Connections

Children's Literature * Bridges recommended titles - # Titles embedded in Bridges Units

*_Can You Count to a Googol_ by Robert E. Wells   *****_How Much is a Million?_ by David Schwartz
*_If You Made a Million_ by David Schwartz   *_Millions to Measure_ by David Schwartz
*_On Beyond a Million_ by David Schwartz
*_Is the Blue Whale the Biggest Thing There Is?_ by Robert E. Wells

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Science
Create and/or use graphs and/or charts generated from simple algorithms to compare alternative solutions to an engineering problem

ELA
CCSS.ELA-LITERACY.SL.4.1

Engage effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with diverse partners on *grade 4 topics and texts*, building on others' ideas and expressing their own clearly.
CCSS.ELA-LITERACY.SL.4.1.A

Come to discussions prepared, having read or studied required material; explicitly draw on that preparation and other information known about the topic to explore ideas under discussion.
CCSS.ELA-LITERACY.SL.4.1.B

Follow agreed-upon rules for discussions and carry out assigned roles.
CCSS.ELA-LITERACY.SL.4.1.C

Pose and respond to specific questions to clarify or follow up on information, and make comments that contribute to the discussion and link to the remarks of others.
CCSS.ELA-LITERACY.SL.4.1.D

Review the key ideas expressed and explain their own ideas and understanding in light of the discussion